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LETTER TO THE EDITOR

# Scaling relation in the burst process of fibre bundles

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**Abstract.** The burst process in a load-carrying bundle of fibres is considered. For a bundle of finite total number of fibres, large bursts near complete failure are shown to exhibit scaling behaviour.

The properties of the distribution of the fibre bundle strengths have been discussed by several authors [1–7]. Consider a bundle of fibres of total number $N$, stretched at both ends. The fibres are assumed to have identical elastic properties, i.e., obey Hooke's law with the same elastic modulus. On the other hand, different strengths are assumed: the maximum load, $t$, the fibres are able to carry before failure has some statistical distribution $p(t)$ for the individual fibres. It was found that [1] for a wide class of threshold strength distributions $p(t)$, the distribution of bundle strengths approaches asymptotically for large $N$ a Gaussian distribution. In their recent preprint [7] Hemmer and Hansen proved that before complete failure the expected number (per fibre) $D(\Delta)$ of bursts with 'size' $\Delta$, in which total number of $\Delta$ broken simultaneously, fits a universal power law

$$D(\Delta) \propto \Delta^{-5/2} \tag{1}$$

for arbitrary distribution of the individual fibre strength. $D(\Delta)$ refers to the distribution in the limit $N \to \infty$. For finite $N$ it cannot, therefore, describe the behaviour of the bursts of very large size, such as the last failure. The motivation of the present letter is to discuss in more detail the failure process for large but finite $N$, and propose another scaling law about the bursts of large size *up to the last failure*.

The failure process consists of many steps of bursts. A typical process is shown in figure 1, where $T$ is the 'time', or the step, at which a burst of size $\Delta$ occurs. The change of $\Delta$ with $T$ is certainly not monotonic. We define $T_k$ and $\Delta_k$ such that the size $\Delta_k$ is the largest one for any $T \leqslant T_k$. The monotonic series $\Delta_k$ is called *the first series*, where the $k$'s are the indices of the series. The complete failure always belongs to the first series of bursts. The time, when the last failure occurs, is taken to be zero, and the corresponding index in the first series is also set zero. The time, $T_k$'s and the indices in the first series, $k$'s, are then always negative or zero. Simulation for any sample gives

$$\cdots, T_{-4}, T_{-3}, T_{-2}, T_{-1}, T_0 = 0$$

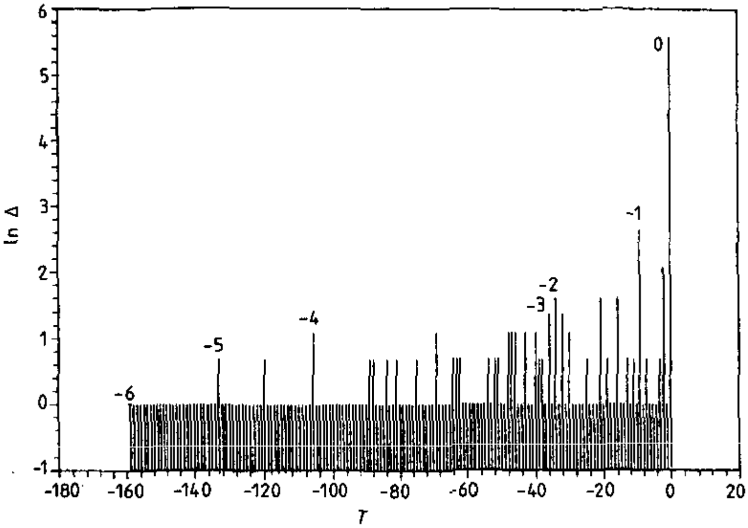



**Figure 1.** The burst process for a bundle with $N = 500$ fibres. This process lasts for 160 steps. The values of index $k$ for the first series are indicated on the graph. $T$ is the time (or step number) of the bursts, and $\Delta$ is the size. The time for the last burst is assumed to be zero.

and

$$\cdots, \Delta_{-4}, \Delta_{-3}, \Delta_{-2}, \Delta_{-1}, \Delta_0$$

where $\Delta_0$ is the size of the last failure. It is found that the value of $\Delta_0$ is, for the uniform threshold distribution, roughly half of the total number of fibres. In figure 2 we show results for $\Delta$ and $T$ of the first series in a simulation experiment.

The threshold strength distribution is taken to be uniform. The data are averaged over $10^3$ samples of bundles with $N = 5\times 10^5$ fibres. There is apparently a power-law relation between $\Delta$ and the index of the series. The standard least square method gives that

$$\Delta_k \propto (k_c - k)^\alpha \tag{2}$$

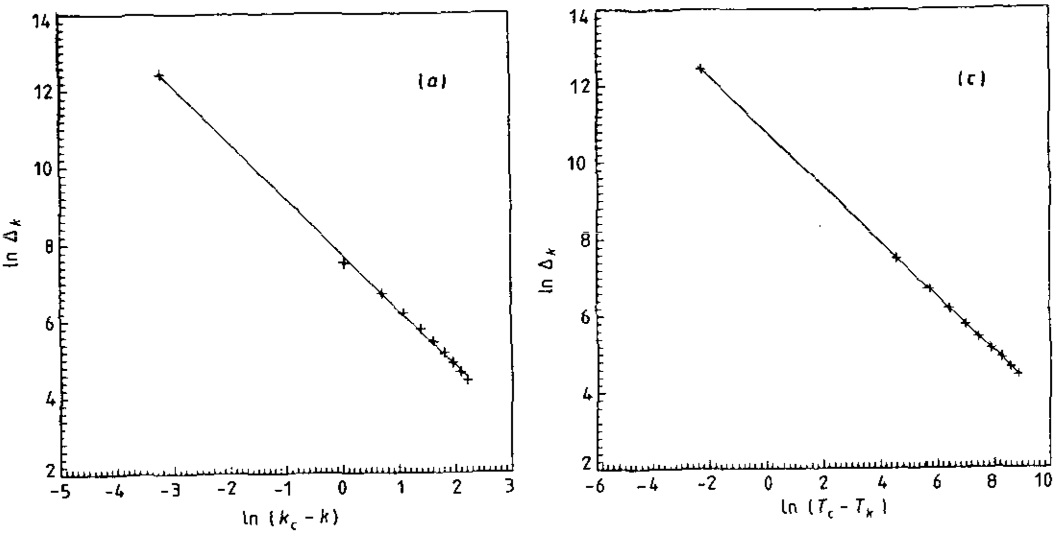
with $\alpha \simeq -1.45$, see figure 2(*a*). In figure 2(*a*) one will find $\ln \Delta_0$ is close to $\ln N/2 = \ln(250\,000) \simeq 12.42$. A similar power-law holds for $T_k$

$$T_c - T_k \propto (k_c - k)^\beta \tag{3}$$

with $\beta \simeq 2.01$, see figure 2(*b*). Then we get the power-law relation between $\Delta_k$ and $T_k$

$$\Delta_k \propto (T_c - T_k)^\nu \tag{4}$$

with $\nu = \alpha/\beta \simeq -0.72$, see figure 2(*c*). For this simulation we have that $k_c \simeq 0.04$ and $T_c \simeq 0.10$. These values, however, are not universal. The fact that the values of $k_c$ and $T_c$ are all positive, and close to zero means that the final burst occurs immediately before the critical time $T_c$. In order to examine the universality of



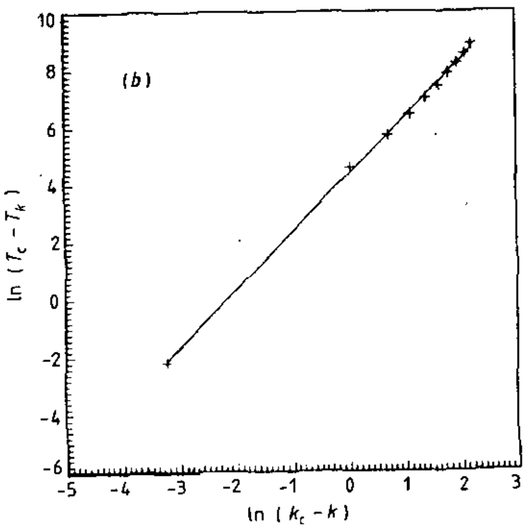




**Figure 2.** The scaling relations for the first series. Uniform threshold strength distribution is assumed. The simulation results are based on $10^3$ samples with $N = 5 \times 10^5$ fibres. (*a*) size $\Delta_k$ versus index $k$; (*b*) time $T_k$ versus $k$; (*c*) $\Delta_k$ versus $T_k$.

this scaling property we chose four other threshold strength distributions below in simulations:

(a) $p(t) = 2(1-t)$ where $0 \leqslant t \leqslant 1$;
(b) $p(t) = 2t$ where $0 \leqslant t \leqslant 1$;
(c) $p(t) = 2|1-2t|$ where $0 \leqslant t \leqslant 1$;
(d) $p(t) = 5t^4 \exp(-t^5)$ where $0 \leqslant t < \infty$.

Within numerical accuracy, the results verify that the values of the exponents $\alpha$, $\beta$ and $\nu$ are universal, independent of the underlying threshold distribution.

Finally we note that the universal behaviour (4) allows prediction of the complete failure. Linear extrapolation of $\Delta_k^{1/\nu}$ versus $T_k$ yields $T_c$, the approximate time for complete failure. It is interesting to note that the phenomenological Gutenberg–Richter power-law for earthquakes [8] is similar to that of fibre failure processes. It is then expected that scaling relations similar to (2)–(4) might be found in earthquake processes.

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